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ANALYSIS OF THE SOUND FIELD OF A LARGE SPAN OSCILLATING
BODY OF REVOLUTION
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UDC 534.231

The theoretical investigation of the spatial sound field produced by an oscillating body of nonzero thickness is a complex problem which has been solved in practice only for a sphere [1].

In this paper an approximate method is proposed for the analysis of the spatial sound field produced by a slender body of revolution with an arbitrary law of its surface oscillation. The solution obtained can be applied to the analysis of the near sound field and the apparent masses of bodies of revolution oscillating in a compressible fluid.

Let us consider the problem of oscillations of a body of revolution in an ideal compressible fluid, which is at rest infinitely far from the body. Let us incroduce the Oxyz Cartesian coordinate system in which the $0 x$ axis is directed along the axis of body symmerry and the origin is at its midsection (see Fig. 1).

Let $S$ be the surface of the undeformed body, $r=\sqrt{y^{2}+z^{2}}, r=R(x)$ is the equation of the generator of the body of revolution, $R_{0}=R(0), Z$ is half the length of the body, $\lambda=$ $Z / R_{0}$ is the span of the body, $\omega$ is the angular frequency of body oscillation, $t$ is the time, $\theta=\arctan (z / y), w(x, \theta, t)$ is the displacement of the body surface along the normal co $S$, $\alpha$ is the speed of sound in the fluid at rest, and $f(x, y, z, t)$ is the velocity potential.

Let us also assume that

$$
\begin{gather*}
\lambda \gg 1, d R / d x \sim R_{0} / l  \tag{1}\\
|w| \ll R_{0}, \quad \partial w / \partial x \sim A / l(A=\max |w|) \tag{2}
\end{gather*}
$$

The assumptions (1) and (2) permit the introduction of two small parameters into the considerations:

$$
\varepsilon_{1}=R_{0} / l, \varepsilon_{2}=A / R_{0}
$$

Let us go over to dimensionless coordinates $x, y, z$ and functions $r, R$ referred to $R_{0}$ by retaining their previous notation. Assuming that the body oscillates according co a given harmonic law for an infinitely long time, we represent the function $w$ and the velocity potential $\varphi$ in the form

$$
\begin{align*}
w(x, \theta, t) & =A \operatorname{Re}\left\{W(x, \theta) \mathrm{e}^{i \omega t}\right\}  \tag{3}\\
\varphi(x, y, z, t) & =a R_{0} \operatorname{Re}\left\{\Phi(x, y, z) \mathrm{e}^{i \omega t}\right\}
\end{align*}
$$

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Fig. 1
The assumption (2) permits solving the problem of determining the velocity potential $\varphi$ outside the oscillating body in an acoustic approximation. In this case the function $\Phi$ satisfies the Helmholtz equation

$$
\begin{equation*}
\Phi_{x x}+\Phi_{y y}+\Phi_{z z}+k^{2} \Phi=0 \quad\left(k=\omega R_{0} / a\right) \tag{4}
\end{equation*}
$$

and the following boundary conditions

$$
\begin{align*}
\nabla \Phi \cdot v= & i k \varepsilon_{2} W(x, \theta) \text { when }(x, y, z) \in S  \tag{5}\\
& \lim _{r_{0} \rightarrow \infty} r_{0}\left(\frac{\partial \Phi}{\partial r_{0}}+i k \Phi\right)=0, \tag{6}
\end{align*}
$$

where $v$ is the normal vector to $S$ and $r_{0}=\sqrt{x^{2}+y^{2}+z^{2}}$.
The expression (5) defines the condition for nonpenetration of the fluid through the surface of the oscillating body, and (6) is the radiation principle. In the general case the function $W(x, \theta)$ has the form

$$
\begin{equation*}
W(x, \theta)=W_{0}(x)+\sum_{n=1}^{\infty}\left[W_{1 n}(x) \cos n \theta+W_{2 n}(x) \sin n \theta\right] \tag{7}
\end{equation*}
$$

Let us first consider axisymmetric body oscillations. In this case,

$$
W(x, \theta)=\dot{W}_{0}(x)
$$

and the solution of (4) can be constructed by using a continuous distribution of sources of some intensity $Q_{0}(x)$ in the segment $|x| \leq \lambda$. The function $\Phi_{0}$ satisfying (4) and the radiation principle (6) then has the form

$$
\begin{equation*}
\Phi_{0}(x, y, z)=\frac{1}{4 \pi} \int_{-\lambda}^{+\lambda} Q_{0}(\xi) \frac{e^{-i r_{k}}}{r_{1}} d \xi \tag{8}
\end{equation*}
$$

where $r_{1}=\sqrt{(x-\xi)^{2}+r^{2}}$.
To determine the desired function $Q_{0}(\xi)$, we have the following integral equation obtained from condition (5) for nonpenetration of the fluid through the body surface:

$$
\begin{equation*}
\frac{1}{\sqrt{1+\left(R^{\prime}(x)\right)^{2}}}\left[-R^{\prime}(x) \frac{\partial \Phi_{0}}{\partial x}+\frac{\partial \Phi_{0}}{\partial r}\right]=i k \varepsilon_{2} W_{0}(x) \text { when } r=R(x) \tag{9}
\end{equation*}
$$

The exact solution of this equation is fraught with serious difficulties. Hence, it is expedient to seek the approximate solution taking account of assumptions (1) and (2).

Let us constrct the approximate solution of (9) by following the idea of the Frankl'Karpovich method developed to solve problems on the flow around slender bodies of revolution by a stationary subsonic gas flow [2].

As the first step, let us find the solution of the corresponding problem of plane gas flow around a circular cylinder being formed at the section $x=$ const $(|x| \leq \lambda)$. The amplitude function of the velocity potential for such a flow $\dot{\Phi}_{0}(x, r)$ can be determined by the formula

$$
\begin{equation*}
\widetilde{\Phi}_{0}(x, r)=\frac{1}{4 \pi} \bar{Q}_{0}(x) \int_{-\infty}^{+\infty} \frac{e^{-i k r_{1}}}{r_{1}} d \xi=-\frac{i}{4} \bar{Q}_{0}(x) H_{0}^{(2)}(k r) \tag{10}
\end{equation*}
$$

where $H_{0}^{(2)}(\mathrm{kr})$ is the Hankel function of the second kind, and the $x$ coordinate plays the part of a parameter.

In conformity with (9), the condition for nonpenetration through a cylinder of the constant radius $r=R(x)$ has the form

$$
\partial \widetilde{\Phi}_{0} / \partial r=i k \varepsilon_{ي} W_{0}(x) \text { when } r=R(x)
$$

It hence follows that

$$
\begin{equation*}
\widetilde{Q}_{0}(x)=4 \varepsilon_{2} W_{0}(x) / H_{1}^{(2)}(k R) \tag{11}
\end{equation*}
$$

Let us note that the boundedness of the first derivatives $R^{\prime}(x)$ and $W^{\prime}(x)$ permits representing the function $\tilde{Q}_{0}(\xi)$ on the segment $|\xi| \leq \lambda$ in the form

$$
\begin{equation*}
\widetilde{Q}_{0}(\xi)=\widetilde{Q}_{0}(x)+F(\xi) \tag{12}
\end{equation*}
$$

where

$$
|F(\xi)| \leqslant|\xi-x| M, \quad M=\sup _{|\xi| \leqslant \lambda}\left|\bar{Q}_{0}^{\prime}(\xi)\right| .
$$

It follows from (11) and conditions (1), (2) that

$$
\begin{equation*}
\bar{Q}_{0}(x) \sim \varepsilon_{2}, \quad \widetilde{Q}_{0}^{\prime}(x) \sim \varepsilon_{1} \varepsilon_{2}, \quad M \sim \varepsilon_{1} \varepsilon_{2} \tag{13}
\end{equation*}
$$

Now let us show that the function $Q_{0}=\tilde{Q}_{0}$ is a solution of (9) in a first approximation.
In place of $Q_{0}$ let us substitute the function $\tilde{Q}_{0}$ defined by (11) into (8). Then, using (12), the function $\Phi_{0}$ can be written in the form

$$
\begin{equation*}
\Phi_{0}(x, y, z)=\widehat{\Phi}_{0}(x, r)+\frac{\widetilde{Q}_{0}(x)}{4 \pi}\left[\int_{-\lambda}^{\infty} \frac{e^{-i} r_{k}}{r_{1}} d \xi+\int_{\infty}^{\lambda} \frac{e^{-i k r_{1}}}{r_{1}} d \xi\right]+\frac{1}{4 \pi} \int_{-\lambda}^{+\lambda} F(\xi) \frac{e^{-i k r_{1}}}{r_{1}} d \xi \tag{14}
\end{equation*}
$$

Let us evaluate the derivatives $\Phi_{0 r}, \Phi_{\alpha x}$ for points of the surface $S$ and let us estimate their order. Using (14), (10), and (12), we have

$$
\begin{gather*}
\left.\Phi_{0 r}\right|_{r=R(x)}=\frac{i k}{4} \bar{Q}_{0}(x) H_{1}^{(2)}(k R)\left\{1-\frac{i R}{\pi k H_{1}^{(2)}(k R)}\left[\int_{-\lambda}^{-\infty} \frac{1}{r_{1}} \frac{\partial}{\partial r_{1}}\left(\frac{e^{-i k r_{1}}}{r_{1}}\right) d \xi+\right.\right. \\
\left.\left.+\int_{\infty}^{\lambda} \frac{1}{r_{1}} \frac{\partial}{\partial r_{1}}\left(\frac{e^{-i k r_{1}}}{r_{1}}\right) d \xi\right]\right\}+\frac{R}{4 \pi} \int_{-\lambda}^{+\lambda} F(\xi) \frac{1}{r_{1}} \frac{\partial}{\partial r_{1}}\left(\frac{\mathrm{e}^{-i k r_{1}}}{r_{1}}\right) d \xi \tag{15}
\end{gather*}
$$

where $r_{1}=\sqrt{(x-\xi)^{2}+R^{2}}$.
Let us estimate the orders of the integrals in this expression by taking into account that for all $|x| \leq \lambda$ the relationships (13) and $R /(\lambda-|x|) \sim \varepsilon_{2}$ are valid.

For the first incegral in the square brackets the following estimate holds:

$$
\begin{gathered}
\left|\int_{-\lambda}^{-\infty} \frac{1}{r_{1}} \frac{\partial}{\partial r_{1}}\left(\frac{e^{-i k r_{1}}}{r_{1}}\right) d \xi\right| \leqslant\left|\int_{-\lambda}^{-\infty}\left(\frac{k}{r_{1}^{2}}+\frac{1}{r_{1}^{3}}\right) d \xi\right|= \\
=\frac{k}{R}\left[\frac{\pi}{2}-\operatorname{arctg} \frac{\lambda+x}{R}\right]+\frac{1}{R^{2}}\left[1-\frac{\lambda+x}{\sqrt{(\lambda+x)^{2}+R^{2}}}\right]= \\
=0\left(\varepsilon_{1} / R\right)+0\left(\varepsilon_{1}^{2} / R^{2}\right) .
\end{gathered}
$$

The second integral in the square brackers has an analogous escimate. Now if it is taken into account that for $|x| \leq \lambda$ the quantity $H_{1}^{(2)}(k R) \sim R^{-1}$, then the second member in the braces is on the order of $\varepsilon_{1}$. As concerns the last member in (15), then

$$
\begin{gathered}
R\left|\int_{-\lambda}^{+\lambda} F(\xi) \frac{1}{r_{1}} \frac{\partial}{\partial r_{1}}\left(\frac{e^{-i k r_{1}}}{r_{1}}\right) d \xi\right| \leqslant R M \int_{-\lambda}^{+\lambda}|\xi-x|\left(\frac{k}{r_{i}^{2}}+\frac{1}{r_{1}^{3}}\right) d \xi= \\
=R M\left\{\frac{k}{2} \ln \left[(\lambda+x)^{2}+R^{2}\right]\left[(\lambda-x)^{2}+R^{2}\right]-2 k \ln R+\frac{2}{R}-\right. \\
\left.\quad-\frac{1}{\sqrt{(\lambda+x)^{2}+R^{2}}}-\frac{1}{\sqrt{(\lambda-x)^{2}+R^{2}}}\right\}=0\left(\varepsilon_{1} \varepsilon_{2} \ln \varepsilon_{1}\right)
\end{gathered}
$$

Therefore,

$$
\begin{equation*}
\left.\Phi_{0 r}\right|_{r=R(x)}=\frac{i k}{4} \widetilde{Q}_{0}(x) H_{1}^{(2)}(k R)\left[1+0\left(\varepsilon_{1}\right)\right]+0\left(\varepsilon_{1} \varepsilon_{2} \ln \varepsilon_{1}\right) . \tag{16}
\end{equation*}
$$

It is convenient to use (8) to calculate the derivative $\Phi_{0}$ by assuming $Q_{0}=\tilde{Q}_{0}$. Then raking account of (12)

$$
\left.\Phi_{0 x}\right|_{r=R(x)}=\frac{\bar{Q}_{0}^{\prime}(x)}{4 \pi} \int_{-\lambda}^{+\lambda} \frac{\mathrm{e}^{-i k r_{1}}}{r_{1}} d \xi+\frac{\widehat{Q}_{0}(x)}{4 \pi} \int_{-\lambda}^{+\lambda} \frac{\partial}{\partial x}\left(\frac{\mathrm{e}^{-i k r_{1}}}{r_{1}}\right) d \xi+\frac{1}{4 \pi} \int_{-\lambda}^{+\lambda} F(\xi) \frac{\partial}{\partial x}\left(\frac{\mathrm{e}^{-i k r_{1}}}{r_{1}}\right) d \xi
$$

Estimating the integrals in this expression, we obtain

$$
\begin{equation*}
\left.\Phi_{0 x}\right|_{r=R(x)}=0\left(\varepsilon_{1} \varepsilon_{2} \ln \varepsilon_{1}\right)+0\left(\varepsilon_{1} \varepsilon_{2} / R\right) \tag{17}
\end{equation*}
$$

Substituting (16) and (17) into condition (9) for nonpenetration of the fluid through the body surface, we obtain

$$
\begin{equation*}
\left.\nabla \Phi_{0} \cdot v\right|_{r=R(x)}=i k \varepsilon_{2} W_{0}(x)+0\left(\varepsilon_{1} \varepsilon_{2} \ln \varepsilon_{1}\right) \tag{18}
\end{equation*}
$$

Therefore, the function $\tilde{Q}_{0}$ is an approximate solution of (4) which differs from the exact solution by the order of magnitude $\varepsilon_{1} \ln \varepsilon_{1}$. Because of the correctness of the boundaryvalue problem (4)-(6), it can be asserted that formula (8) for $Q_{0}=Q_{0} y i e l d s$ an approximate solution in the whole flow domain which differs slightly from the exact solution.

Now let the body of revolution oscillate according to the arbitrary law (7) under the assumptions (1) and (2). In this case the solution of the boundary-value problem (4)-(6) can be constructed by a linear combination of multidipoles of definite type. The cases of even and odd dependences of the function $W(x, \theta)$ on the variable $\theta$ should hence be distinguished.

Let us first construct the solution for even functions $W(x, \theta)$. Because of linearity of the problem it is sufficient to find the solution for the case

$$
\begin{equation*}
W(x, \theta)=W_{1 n}(x) \cos n \theta \tag{19}
\end{equation*}
$$

Let us note that the function $\cos n \theta$ can be represented by an expansion in powers of cos $\theta$ in the form [3]

$$
\begin{equation*}
\cos n \theta=2^{n-1} \cos ^{n} \theta \frac{1}{1} \sum_{j=1}^{n}(-1)^{j} \frac{n}{2 j} C_{n-j-1}^{j-1}(2 \cos \theta)^{n-2 j} \tag{20}
\end{equation*}
$$

where $C_{n}^{m}$ are the binomial coefficients.
In conformity with (20), the solution of the boundary-value problem (4)-(6), (19) can be sought by using a linear combination of multidipoles of the type

$$
Q_{1 m}(\xi) \frac{\partial^{m}}{\partial y^{m}}\left(\frac{\mathrm{e}^{-i k r_{1}}}{r_{1}}\right) \quad(m=0,1, \ldots, n)
$$

distributed continuously along the $O x$ axis for $|\xi| \leq \lambda$. Denoting this solution in terms of $\Phi_{1}(x, y, z)$, we have

$$
\begin{equation*}
\Phi_{1}=\sum_{m=0}^{n} \Phi_{1 m}, \quad \Phi_{1 m}=\frac{1}{4 \pi} \int_{-\lambda}^{+\lambda} Q_{1 m}(\xi) \frac{\partial^{m}}{\partial y^{m}}\left(\frac{e^{-i k r_{1}}}{r_{1}}\right) d \xi \tag{21}
\end{equation*}
$$

The functions $Q_{1 m}$ are determined from the nonpenetration condition (5), which in the case under consideration has the form

$$
\begin{equation*}
\frac{\partial \Phi_{1}}{\partial v}=i k \varepsilon_{2} W_{1 n}(x) \cos n \theta \quad \text { for } \quad(x, y, z) \in S \tag{22}
\end{equation*}
$$

Substituting (21) into condition (22) and raking into account that $y=r \cos \theta$, and cos $n \theta$ is expressed in terms of powers of $\cos \theta$ by using (20), we arrive at a system of integral equations in the functions $Q_{1} m(m=0, \ldots, n)$. We solve the system obtained approximately by using the method proposed above. Following this method, we find the solution of the corresponding boundary-value problem of plane-parallel fluid flow around an infinite cylinder with radius $r=R(x)$, where $x$ plays the part of a parameter. The amplitude function of the velocity potential for such a flow $\tilde{\Phi}_{1 m}$, produced by a multidipole with intensity $\tilde{Q}_{1 m}(x)$, is determined by the formula

$$
\widetilde{\Phi}_{1 m}(x, r, \theta)=-\frac{i}{4} \bar{Q}_{1 m}(x) \frac{\partial^{m}}{\partial y^{m}} H_{0}^{(2)}(k r)
$$

where $r=\sqrt{y^{2}+z^{2}}$.
Let us require that

$$
\partial \Phi_{1 m} / \partial v=\partial \widetilde{\Phi}_{1 m} / \partial r \quad \text { for } \quad r=R(x)
$$

Then by analogy with the deduction of the relationship (18), it can be shown that

$$
\begin{equation*}
Q_{1 m}=Q_{1 m}+0\left(\varepsilon_{1} \varepsilon_{2} \ln \varepsilon_{1}\right), \tag{23}
\end{equation*}
$$

while the function $\tilde{Q}_{1 m}$ is a quantity on the order of $\varepsilon_{2}$.
Now substituting (21), in which $\Phi_{1 m}=\tilde{\Phi}_{1 m}$, into the boundary condition (22), we obtain a linear system of algebraic equations to determine the quantities $\tilde{Q}_{1 m}$ ( $m=0, \ldots, n$ ). Furthermore, replacing the functions $Q_{1 m}$ in (21) by $Q_{1 m}$, we obtain the desired approximate solurion of the problem (4)-(6) for the case of body oscillations according to the law (19).

The solution of the problem (4)-(6) is constructed by an analogous means for body oscillations according to the law

$$
\begin{equation*}
W(x, \theta)=W_{2 n}(x) \sin n \theta . \tag{24}
\end{equation*}
$$

Taking into account that [3]

$$
\sin n \theta=\sin \theta \sum_{j=1}^{n}(-1)^{j+1} C_{n-j}^{j}(2 \cos \theta)^{n-2 j+1},
$$

the solution $\Phi_{2}$ of this problem can be sought in the form

$$
\begin{equation*}
\Phi_{2}=\sum_{m=1}^{n} \Phi_{2 m}, \quad \Phi_{2 m}=\frac{1}{4 \pi} \int_{-\lambda}^{+\lambda} Q_{2 m}(\xi) \frac{\partial}{\partial z}\left\{\frac{\partial^{m-1}}{\partial y^{m-1}}\left(\frac{e^{-i k r}}{r_{1}}\right)\right\} d \xi . \tag{25}
\end{equation*}
$$

The approximate solution of the problem (4)-(6) is constructed by the same means. In particular, the amplitude function $\Phi_{2 m}$ of the corresponding plane-parallel flow around the cylinder $r=R(x)$ is determined by the formula

$$
\widetilde{\Phi}_{2 m}(x, r, \theta)=-\frac{i}{4} \widetilde{Q}_{2 m}(x) \frac{\partial}{\partial z}\left\{\frac{\partial^{m-1}}{\partial y^{m-1}}\left(H_{0}^{(2)}(k r)\right)\right\},
$$

and the functions $\tilde{Q}_{2 m}$ by the solution of a system of algebraic equations obtained from the nonpenetration condition.

It can again be shown that $\tilde{Q}_{2 m}$ is an approximate solution for $Q_{2 m}$ with an estimate of the type (23).

Within the framework of the acoustic approximation, the hydrodynamic pressure $p$ in a fluid produced by the oscillating body is determined by the Cauchy-Lagrange integral

$$
\begin{equation*}
p-p_{\infty}=-\rho \partial \varphi / \partial t, \tag{26}
\end{equation*}
$$

where $p_{\infty}$ is the pressure in the fluid at rest and $\rho$ is the fluid density.
Let us introduce the dimensionless pressure function $C_{p}(x, y, z, t)$ by assuming

$$
\begin{equation*}
p-p_{\infty}=(1 / 2) \rho a^{2} C_{p} . \tag{27}
\end{equation*}
$$

In conformity with expressions (26), (27), and (3)

$$
\begin{equation*}
C_{p}=-2 k \operatorname{Re}\left\{i \Phi^{i \omega t}\right\} . \tag{28}
\end{equation*}
$$

Let us clarify the asymptotic of the solution in some direction given by the unit vector $l=\left(z_{x}, z_{y}, l_{z}\right)$. Let us use the notation

$$
r=\sqrt{y^{2}+z^{2},} r_{0}=\sqrt{x^{2}+y^{2}+z^{2}}, r_{1}=\sqrt{(x-\xi)^{2}+y^{2}+z^{2}} .
$$

Assuming $|\xi| \leq \lambda, r_{0} \gg \lambda$, we have

$$
r_{1}=r_{0}-\frac{x \xi}{r_{0}}+0\left(r_{0}^{-1}\right), \quad \frac{1}{r_{1}}=\frac{1}{r_{0}}+0\left(r_{0}^{-2}\right)
$$

It hence follows that

$$
\begin{equation*}
\frac{\mathrm{e}^{-i k r_{1}}}{r_{1}}=\frac{\mathrm{e}^{-i k r_{0}}}{r_{0}}\left[\mathrm{e}^{i k l_{x}^{\xi}}+0\left(r_{0}^{-1}\right)\right] . \tag{29}
\end{equation*}
$$

Substituting (29) into (8), we obtain

$$
\begin{equation*}
\Phi_{0}(x, y, z)=\frac{\mathrm{e}^{-i_{k} r_{0}}}{4 \pi r_{0}} \int_{-\lambda}^{+\lambda} Q_{0}(\xi) \mathrm{e}^{i \hbar l^{\Sigma} \bar{\xi}} d \xi+0\left(r_{0}^{-2}\right) \tag{30}
\end{equation*}
$$

Formulas (28) and (30) determine the asymptotic of the solution in the direction of the vector $Z$ for the case of axisymmetric body oscillations.

To obtain asymptotic formulas in the case of body oscillations according to the laws (19) and (24), it should be kept in mind that

$$
\begin{gathered}
\frac{\partial^{m}}{\partial y^{m}}\left(\frac{e^{-i k r_{0}}}{r_{0}}\right)=(-i k)^{m}\left(\frac{y}{r_{0}}\right)^{m} \frac{\mathrm{e}^{-i k r_{0}}}{r_{0}}+0\left(r_{0}^{-2}\right) \\
\frac{\partial}{\partial z} \frac{\partial^{m-1}}{\partial y^{m-1}}\left(\frac{\mathrm{e}^{-i k r_{0}}}{r_{0}}\right)=(-i k)^{m} \frac{z}{r_{0}}\left(\frac{y}{r_{0}}\right)^{m-1} \frac{\mathrm{e}^{-i k r_{0}}}{r_{0}}+0\left(r_{0}^{-2}\right) \\
z / r_{0}=\sqrt{1-l_{x}^{2}} \sin \theta, \quad y r_{0}=1-\overline{1-l_{x}^{2}} \cos \theta
\end{gathered}
$$

Taking these expressions into account, (21) and (25) for the amplitude functions $\Phi_{1}, \Phi_{2}$ can be represenced for $r_{0} \gg \lambda$ in the form

$$
\begin{gather*}
\Phi_{1}(x, y, z)=\frac{\mathrm{e}^{-i k r_{0}}}{4 \pi r_{0}} \sum_{m=0}^{n}\left(-i k \sqrt{1-i_{x}^{2}} \cos \theta\right)^{m} \int_{-\lambda}^{+\lambda} Q_{1 m}(\xi) \mathrm{e}^{i k l x^{\frac{t}{y}} d \xi \div 0\left(r_{0}^{-2}\right) ;}  \tag{31}\\
\Phi_{2}(x, y, z)=\frac{\mathrm{e}^{-i k r_{0}}}{4 \pi r_{0}} \sin \theta \sum_{m=1}^{n}\left(-i k \sqrt{1-l_{x}^{n}}\right)^{m} \cos ^{m-1} \theta \int_{-\lambda}^{+!} Q_{2 m}(\xi) \mathrm{e}^{i h_{x} n_{x}} d \xi \div 0\left(r_{0}^{-2}\right) . \tag{32}
\end{gather*}
$$

It follows from (31) and (32) that in the case of low-frequency oscillations of a body, when the parameter is $k=\omega R_{0} / a \ll 1$, the main contribution to the sound field far from the oscillating body is due to its axisymmetric oscillations.

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SHOCK LOADING OF AN INFINITE PLATE CONTIGUOUS TO A FLUID

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UDC 532.5:539.37

Questions of the effect of shock loadings on infinite plates in contact with a fluid have been examined in a number of papers [1-9]. The axisymmetric deformation of plates was studied in [1-6], while [1, 7-9] were devoted to the plane problem. The investigations were execured in different formulations. Different kinds of plate loadings (the effect of acoustic pressure waves, concentrated forces or distributed loads; assignment of the motion velocity) were considered. The plate deformation was described by different equations (the membrane deflection equation, the Bernouli-Euler bending equation, or a Timoshenko-type equation). The main method of solving these problems is the method of integral transforms. Definite difficulties occur during the solution in going from the transforms to the originals. Still greater difficulties are encountered in analyzing the solution and obtaining specific numerical results in the originals written in the form of complex single or double integrals. The solution in a number of papers $[3,5,8]$ is hence constrained to the writing of formulas in quadratures, while the problem is solved in other investigations [1, 2, 4, 6, 9] by asymptotic methods which are valid in a definite range of time variation. There are also separate results obtained by using the numerical inversion of the Laplace transform in [6] which is devoted to the effect of a spherical pressure wave.

In this paper, the solution of the plane problem of bending an infinite plate in contact with a compressible fluid occupying a half-space along one of the sides of the plate is sought by using integral transforms.
§1. The $X, Z$ coordinate $p l a n e$ is in the plane of the plate, and the' $Y$ axis is directed into the fluid. A transverse load distributed uniformly along the $Z$ axis is applied instantaneously to the plate along this whole axis. It is sufficient to consider the motion in one

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